The Distribution of Wealth and the Assignment of Control Rights in the Firm

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Abstract

We explore the relationship between wealth and the assignment of control rights in the firm by modeling two distinct ways of governing team production: conventional and employee-run. Our approach is novel in assuming that both labor and credit markets are subject to agency problems based on asymmetric information: employees have more accurate knowledge of their level of work effort than employers, and borrowers have more accurate knowledge of the value of the projects they undertake than do the lenders who finance these projects. We argue that the conventional firm generally has a comparative advantage in handling agency problems regarding lending and borrowing, whereas the employee-run firm's advantage generally lies in handling agency problems regarding the intensity and quality of work.

We show that where both types of firms coexist, the equilibrium fraction of workers in employee-run firms and the distribution of wealth are mutually determining. One implication is that efficiency considerations alone are insufficient to explain the assignment of control rights in firms, since an exogenous increase in worker wealth supports a higher fraction of workers in employee-run firms.

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1   Introduction

By an *employee-run* firm we mean an enterprise whose management is accountable the firm's employees. A *conventional* firm is one whose management is accountable to owners of the firm's capital assets, who are distinct from the firm's employees.[1] In this paper we show that under plausible conditions, where both types of firms coexist, and where agents allocate themselves among firms to maximize the present value of their utility, the equilibrium fraction of workers in employee-run firms and the distribution of wealth are mutually determining.

We argue below that incentive problems concerning work effort may be attenuated by assigning residual claimancy and control rights to workers themselves. Moreover there is substantial evidence consistent with the superior productivity of employee-ownership.[2] Why then are employee-run firms not more widely observed in market economies?[3] In this paper we suggest at least a partial explanation of this phenomenon.[4] We show that under plausible informational assumptions, the limited wealth of workers renders employee-ownership unattractive. As *prima facie* evidence for this position, we note that where capital requirements are low and employees are relatively wealthy, employee-run firms are common, as in consulting and accounting firms, law offices, and group medical practices (**?**):66–88.[5]

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[1]Since we assume in this paper that all firm members are identical, the precise nature of the aggregation of individual choices in the employee-run firm is unimportant. Moreover, the administrative structure of the employee-run firm may or may not differ from that of its conventional counterpart. Indeed, we here ignore difference in organizational structure across different types of firms.

[2]See **?**) and **?**). The plywood coops described by Craig and Pencavel are a reasonably close approximation to the employee-run and owned firms described in this paper. The extent of employee ownership and participation in managerial decision-making in Kruse and Blasi's sample, by contrast, is quite limited. See also **?**), **?**), **?**), **?**), **?**), and (**?**, **?**). For a recent review of the literature, see **?**).

[3]**?**) and **?**) are the standard survey of responses to this question. See also **?**) and **?**). (**?**) influential answer to this problem is that an efficient contract will assign residual claimancy and control rights to an owner/manager who monitors team members. However, since most production in advanced economies takes place in firms in which residual claimancy and control is held by owners who are neither monitors nor producers, in principle it should be no more difficult for worker-owners than for traditional owners to purchase effective monitoring services. Hence the relevance of our analysis.

[4]For complementary treatments, see **?**), **?**), and **?**).

[5]If the only drawback to the employee-run firm is the lack of wealth of its members, it may be asked why wealthy agents do not come together to form firms that can compete successfully against conventional firms. Doubtless the main reason is most of the jobs employees hold are considered undesirable by agents whose wealth is sufficient to allow

We conclude that the distribution of wealth and the assignment of control rights are mutually determining. The fraction of workers in employee-run firms increases with worker wealth because wealthier workers are able to hold more equity in their firm, and hence have lower failure rates and pay lower interest rates on borrowed funds. Worker wealth increases with the fraction of workers in employee-run firms because employee ownership increases the worker's exposure to risk, and workers will be willing bear this increased risk only if it is associated with a higher expected return. Both the higher return and the increased risk exposure foster an increase in worker saving, which entails an increase in the expected lifetime wealth of workers.

Our model addresses two types of asymmetric information: a worker has better knowledge of the intensity and quality of the work he performs than does his employer, and a borrower has better knowledge of the value of a project he undertakes than does the lender who helps finance this project. The employer (both in conventional and employee-run firms) must thus induce the worker to perform without assuming the worker will truthfully report the intensity and quality of his effort, and the lender must induce the borrower to repay without relying on the borrower' honesty in reporting the firm's financial status and prospects. Employers traditionally solve the effort problem by monitoring employee performance and using the threats of dismissal and promises of advancement to evince a high level of worker effort.[6] Traditional capital markets solve the repayment problem by requiring borrowers to share in the equity of their projects, and by using debt contracts that automatically trigger a default situation when a loan is not repaid on time.[7]

But the effectiveness of these mechanisms depends on the institutional contexts within which they operate and the wealth of the parties to the exchange. In our model of the employee-run firm, workers are residual claimants. For reasons discussed below, this gives the employee-run firm a relative advantage in eliciting work effort. By contrast, the owners of a conventional firm tends to be wealthy and hence to hold considerable equity in their projects, allowing them to obtain credit on favorable terms. Equity finance in the conventional firm attracts highly diversified stockholders, which also reduces the firm's financing costs.

Our model differs from the existing literature on employee-run firms by

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them to form employee-run firms.

[6] See ?), ?), ?), ?) and ?).

[7] The informational conditions under which debt contracts are optimal are described in ?).

considering asymmetric information in both labor and capital markets.[8] By focusing on the difference in the way the two types of firm handle market failures arising from incomplete contracts, our approach contrasts with 'perfect market' models, which yield the implausible result that in competitive equilibrium the location of residual claimancy and control rights in the two types of firm is of no consequence, and that under suitable conditions both implement Pareto optimal allocations (**?**, **?**).[9]

Our model also differs from the common practice in the literature of imposing arbitrary restrictions on the types of property holdings permitted in the employee-run firm—requiring, for example, that its assets be held as common property by its members.[10] On the basis of such assumptions one can show, for instance, that the employee-run firm invests too little and exhibits a perverse supply response to output price variations. But since these restrictions on property holdings are generally Pareto suboptimal, the conclusion that the employee-run firm is an inefficient alternative to the conventional firm is not surprising. Indeed, **?**), **?**) and others have shown that these problems disappear when members of the employee-run firm are permitted to own negotiable shares that may be sold to new members upon leaving the firm. Our approach, by contrast, focuses on differences in contractual form that are inherent in the informational asymmetries of capital and labor markets.[11]

An implication of our analysis is that the assignment of property rights in an economy cannot be explained in terms of efficiency considerations alone, since the pattern of control depends on the distribution of wealth.[12]

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[8]For previous research, see **?**), **?**), **?**), and the works summarized in **?**).

[9]While our agency-theoretic approach can be defended on grounds of realism, the same cannot be said for our assumption that work team members are identical, by which we deliberately avoid consideration of a possibly costly collective choice problem facing the employee-run firm, namely how to arrive at decisions. See, for instance, **?**), **?**), and **?**).

[10]See for instance **?**),**?**), **?**), **?**), **?**), **?**), and for an insightful review of the literature, **?**).

[11]In fact, to limit the complexity of our model, we do assume members of the employee-run firm have access to debt but not external equity finance. For a treatment of the disadvantages of the employee-run firm when external equity finance is available, see Gintis (**?**, **?**). We also assume that the stochastic element in the returns to the firm cannot be altered by the firm's principals. For a model in which the level of project risk is endogenous, see **?**).

[12]Several authors have suggested that the assignment of residual claimancy and control in market economies can be deduced from issues of efficiency alone, including **?**), **?**), **?**), **?**), and **?**). Analyses in which the assignment of residual claimancy and control depends on the distribution of wealth include **?**), **?**), **?**), **?**), **?**), and **?**).

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2 THE EMPLOYMENT RELATIONSHIP

We assume a production process that requires capital investment $\mathbf{k}$ and n worker-hours per period. The conventional firm has an owner/employer and a homogeneous labor force, with revenue schedule $q(l + \epsilon)$, where l is the total effort supplied to the production process, ϵ is a random variable with zero mean, and q is the price of the output.[13] If e^c (the ^{c} stands for 'conventional') is the work effort per hour provided by a typical employee, then $l = ne^c$. Net revenue in any period is then given by

$$\pi = q(ne^c + \epsilon) - n(w + m) - r^c \mathbf{k},$$

where w and m are the wage and monitoring expenditures per worker-hour, r^c is the interest rate paid on the capital (which for simplicity we assume does not depreciate). We assume that individual worker effort is observable by the owner only imperfectly and at the fixed cost m per worker-hour, and aggregate effort cannot be verifiably inferred from gross revenue due to the presence of the error term ϵ. For simplicity, we use the normalizations $n = 1$ and $\mathbf{k} = 1$. Thus the above equation becomes

$$\pi = qe^c - (w + m) - r^c + \epsilon. \tag{1}$$

We assume the firm has an exogenously given and fixed probability $\gamma > 0$ of failure in any period, which we interpret as a shift in the revenue schedule rendering the firm thenceforth worthless. In this case any firm revenues are dissipated in liquidation and the capital stock becomes worthless. We write the probability of firm success in any period as $p^c = 1 - \gamma$. We also assume the owner/employer is risk-neutral and is indifferent between investing in risk-free bonds at the interest rate ρ, and investing in the firm if the expected rate of return is ρ. For this to be the case, we must have $(1+\rho) = (1-\gamma)(1+r^c)$, since with probability γ the owner/employer receives nothing. Thus we have

$$r^c = \frac{\rho + \gamma}{1 - \gamma}.$$

In dealing with an employee, the owner/employer is the principal in an infinite-horizon repeated principal-agent relationship, and maximizes the expected value of the firm's net revenue. The owner/employer does not observe the employee's choice of effort e^c, but does observe a signal generated by a monitoring device that is correlated with e^c. If the signal indicates

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[13]These simplifying assumptions play no substantive role below.

insufficient effort, the employee is dismissed. Specifically, the employer's monitoring strategy gives rise to a probability of dismissal schedule $f^c(e^c)$, where $df^c/de < 0$ (i.e., higher work effort reduces the probability of dismissal).[14] We assume this schedule is known to the employees.

The employee takes the wage w, the dismissal probability schedule $f^c(e^c)$ and the probability of firm success p^c as given, and chooses e^c to maximize the present value of future utility. To derive the employee's best response function, suppose the employee has a twice differentiable utility function $u(w, e^c)$, increasing in the wage w, decreasing in effort e^c, strictly concave in w and strictly convex in e^c. The employee then maximizes the expected present value of utility v^c over an infinite horizon which, assuming that flows accrue at the end of the period, is

$$v^c(w) = \frac{u(w, e^c) + p^c(1 - f^c(e^c))v^c(w) + (1 - p^c + p^c f^c(e^c))z}{1 + \rho}, \quad (2)$$

where ρ is the employee's rate of time preference (assumed for simplicity to equal the risk-free interest rate), $1 - p^c + p^c f^c(e^c)$ is the probability of job loss (from both dismissal and firm bankruptcy), and z is the present value of the employee's stream of utility should the job be terminated. We take z as exogenously given.[15] Rewriting (??), we have

$$v^c = \frac{u(w, e^c) - \rho z}{\rho + 1 - p^c + p^c f^c(e^c)} + z, \quad (??a)$$

which says that the value of job tenure $v^c - z$ is the net flow of utility from having the job, $u(w, e^c) - \rho z$, discounted by the sum of the rate of time preference, the probability of bankruptcy, and the probability of dismissal.

Maximizing v^c gives the first order condition[16]

$$u_e = (v^c - z)p^c f_e^c(e^c), \quad (3)$$

which defines a best response function $e^c = e^c(w)$. We interpret equation (??) as requiring that the team member increase effort e^c until the marginal disutility of effort u_e offsets the marginal effect of additional effort

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[14]The dismissal probability schedule $f^c(e^c)$, which we take as given, can be derived as a solution to an optimal control problem. Similarly the optimal level of monitoring resources m, which we treat as fixed, can be determined endogenously. See ?) and ?).

[15]In a more complete model, z would depend on the cost of job search, the expected duration of unemployment, the level of unemployment compensation, and the value of leisure. See ?), ?), and ?).

[16]In expressions like u_e, we are using a subscript to a function to represent the partial derivative of the function with respect to the variable subscript.

on the probability of retaining the job $f_e^c(e^c)$ multiplied by the product of the job rent $v^c - z$ and the probability of firm survival p^c, retaining the job being worthless if the firm fails.

Equation (**??**) gives rise to a best response function $e^c = e^c(w)$, which can easily be shown to be increasing in w and decreasing in z in a neighborhood of firm equilibrium. The employer maximizes profits by choosing the wage rate w, subject to the employee best response function $e^c(w)$, thus determining equilibrium effort e^c, the dismissal probability $f^c(e)$, and the present value of employment v^c.[17]

The employee-run firm differs from the conventional firm in three ways. First, the employees are residual claimants and share equally the net revenue of the firm. Second, the employees hold control rights in the firm assets, and thus may direct and remove the firm's management. Third, we assume workers are not wealthy and are risk-averse, so they finance the firm's capital expenditure in part by acquiring debt. For ease of comparison, we assume that the disutility of labor and the termination-based labor discipline system are common to both types of firm.

These assumptions imply that the only variable to be determined as a group by the worker/members in the employee-run firm is the amount of borrowing.[18] As in the conventional firm, workers in the employee-run firm choose effort levels individually to maximize the present value of being in the firm. Knowing their own and other workers' best response functions, employees as a group direct the firm's managers to select a level of external debt that maximizes the value of membership in the firm.

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[17]Under plausible conditions $v^c > z$ in equilibrium, so the employee receives an employment rent, the maintenance of which induces an effort level above that which would be forthcoming in the absence of a threat of dismissal. For examples of such models, see **?**), **?**), and **?**).

[18]While we take the capital intensity and the monitoring intensity m as fixed and hence unvarying across types of firms, there is some evidence that worker-owned firms choose lower capital and monitoring intensity than their conventional counterparts (**?**, **?**). The choice of lower capital intensity is a plausible response to the employee-run firm's less advantageous access to credit markets, and the choice of lower monitoring intensity is reasonable since both payments to labor and monitoring expenditure are both expenses for the conventional firm, but payments to labor are not an expense for the employee-run firm. Moreover, if the employee-run firm has higher capital costs than the conventional firm and its members are risk-averse (as is the case in our model), the employee-run firm may well choose a lower capital/labor ratio and a less variable stream of future revenues than the conventional firm. When such tendencies are strong, and if the increased employment associated with the more labor-intensive technology is not highly valued, the efficiency gains of the employee-run firm analyzed in this paper may be nullified or even reversed. We abstract from these effects here to focus on the debt/equity decision.

Suppose the employee-run firm's probability of success is p^l, and a worker with wealth W who contributes equity k to the firm receives a rate of return on this equity $r(k, p^l, W)$ that leaves the worker just indifferent between being holding a fully diversified portfolio of size W on the one hand and holding k in the firm's equity and $W - k$ in a fully diversified portfolio on the other.[19] We consider the income stream of the worker as having two parts, a non-stochastic claim y on the net revenue of the firm—the counterpart to the wage in the conventional firm—and a stochastic return on the capital k invested with expected return $r(k, p^l, W)$. The value of membership is defined analogously to (??a), so for a given y the member determines individual effort e^l by solving

$$v^l(y) = \max_{e^l} v^l = \frac{u(y, e^l) - \rho z}{\rho + f^l(e^l)} + z, \qquad (4)$$

where $f^l(e^l)$ is the worker's probability of job loss in the employee-run firm, including dismissal and firm failure (the l here stands for 'labor'). This gives rise to a member's best response function $e^l = e^l(y)$. The members instruct the manager to solve

$$\max_k v^l(y) = \frac{u(y, e^l(y)) - \rho z}{\rho + f^l(e^l(y))} + z,$$

subject to the feasibility constraint

$$y = q e^l(y) - m - r^l(k)(1 - k) - r(k, p^l, W)k, \qquad (5)$$

where $r^l(k)$ is the interest rate on the external debt $1 - k$, and $r(k, p^l, W)$ is the interest rate offered to equity capital k contributed by the member with wealth W and probability of firm success p^l. In the next section we derive the schedules $r^l(k)$ and $r(k, p^l, W)$, assuming that $r(k, p^l, W)$ just renders the member indifferent between investing k in the firm and holding an optimally diversified portfolio.

If the employee-run firm exhibits greater effort for given arguments of the member's best response function than does the conventional firm, i.e., if $e^l(y) > e^c(w)$ with $y = w$ for some range of parameter values for w and y, we say the employee-run firm *dominates* the conventional firm in regulating effort over this range.

We offer the following reason to believe that the employee-run firm might dominate the conventional firm in regulating effort. The distribution of

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[19]Wealth W does not include the present value of job utility v^l.

residual claimancy and control rights that characterize the employee-run firm alters the problem of enforcing labor discipline, not by eliminating agency problems, but by providing the employee-run firm with monitoring mechanisms unavailable or more costly to the conventional firm. Employees in both types of firm frequently have low cost access to information concerning the work activities of fellow workers, and in the employee-run firm each has an interest in the effort levels other workers. The residual claimancy status of workers in the employee-run firm thus provides a motive not for one's own effort, but for *mutual monitoring*. Our model thus assumes that the size of the work team is sufficiently large that individual members do not take into account the effects of their choice of effort level on profits or the probability of firm failure, but that the costs of mutual monitoring are sufficiently low that team members are willing to monitor the activity of others, given their residual claim on the firms income.

Several cogent models of mutual monitoring in teams support this reasoning.[20] The basic intuition of such models is that the close and durable interactions among members of a work team, combined with residual claimancy, provide members with both opportunities and incentives to punish shirkers either directly or by facilitating the work of the monitor.

The model we present here has the advantages of being extremely simple and depending on an aspect of human behavior widely verified in experimental studies, yet absent in standard utility functions. We call this the *principle of reciprocity*: human agents willingly help others who have helped them, and willing hurt others who have hurt them, even when such action involves a personal cost to the agent, net of any personal benefits other than the utility derived from the act itself.[21]

For a worker in the employee-run firm that chooses effort level e^l, let $x(e^l)$ be the expected number of coworkers who observe him shirking, where $x(e^l)$ is a decreasing function. Suppose each worker who observes a coworker shirking experiences a disutility d^l from being deceived, and experiences a utility h^l from applying a moral or material sanction of some sort that imposes a disutility s^l to the shirker. If workers are equally likely to observe an instance of shirking, then each worker will expect to see $x(\bar{e}^l)$ instances of

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[20]Models of peer monitoring in cooperatives have been analyzed by ?) ?), ?), ?), ?), ?), ?), and ?).

[21]The principle of reciprocity is probably best known to economists from ?). For other treatments, see ?) and ?), and for experimental evidence and bibliographic references, see ?) and ?). In a series of experiments, Ernst Fehr and his collaborators have repeatedly shown that people are willing to pay to punish those who, in their eyes, have treated them poorly (?).

shirking (since we have normalized the number of firm members at unity), where $\bar{e}^l$ is the expected effort level of other workers. A given worker then maximizes

$$v^l = \frac{u(y, e^l) - (d^l - h^l)x(\bar{e}^l) - s^l x(e^l) - \rho z}{\rho + f^l(e^l)} + z, \qquad (??a)$$

giving rise to a worker best response function

$$e^l(y) = e^l(y; d^l, h^l, s^l, \bar{e}^l)$$

that reduces to (??) when $d^l = h^l$ and $s^l = 0$. We first show that an exogenous increase in s^l shifts up the $e^l(y)$. The first order condition corresponding to (??a) is

$$v_e^l = \frac{u_e - sx_e^l - f_e^l(v^l - z)}{\rho + f_e^l} = 0.$$

Differentiating totally with respect to s^l, we get

$$v_{ee}^l \frac{de^l}{ds^l} + v_{es}^l = 0.$$

But $v_{ee}^l < 0$ by the second order condition, and $v_{es}^l = -x_e/(\rho + f^l) > 0$. Hence $de^l/ds^l > 0$. Hence *ceteris paribus*, an increase in the willingness of workers to penalize one another for shirking increases the amount of effort elicited by the employee-run firm. However a similar argument shows that an exogenous increase in the term $(d^l - h^l)\bar{e}^l$ reduces equilibrium work effort. Hence for mutual monitoring to be effective, the subjective cost of being caught shirking and the expected number of coworkers applying moral sanctions must be large compared to the net subjective cost of applying sanctions at the expected rate of shirking. We assume this is so in what follows.[22]

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[22] The reader will note that this mutual monitoring mechanism is impervious to two standard objections to mutual monitoring models. First, why cannot the conventional firm mimic the employee-run firm, implementing the latter's incentive scheme? In our case this is impossible, because if a worker is not a residual claimant, he will not interpret shirking on the part of a coworkers as a defection against himself ($d^l = 0$), and will not retaliate. Of course the conventional owner could confer residual claimancy upon the employees, but this would represent an incentive incompatible separation of residual claimancy and control. Second, why cannot coworkers in a mutual monitoring scheme simply lie about their coworkers' performance, for personal reasons unconnected with the operation of the enterprise? In our model this is unmotivated, since a coworker receives the utility h^l only if in fact the coworker shirked.

On the above assumptions, the assignment of residual claimancy to team members is thus equivalent to a technical improvement in the monitoring technology, allowing a more accurate signal of worker intensity for given monitoring resources. We assume in the rest of the paper that for this reason the employee-run firm dominates the conventional firm in regulating effort over the appropriate range of monetary remunerations and monitoring levels. This situation is depicted in Figure **??**, which illustrates the fact that for a given worker payment (wage in the conventional firm and residual share in the employee-run firm) the effort forthcoming is greater in the employee-run firm for a given level of monitoring resources.
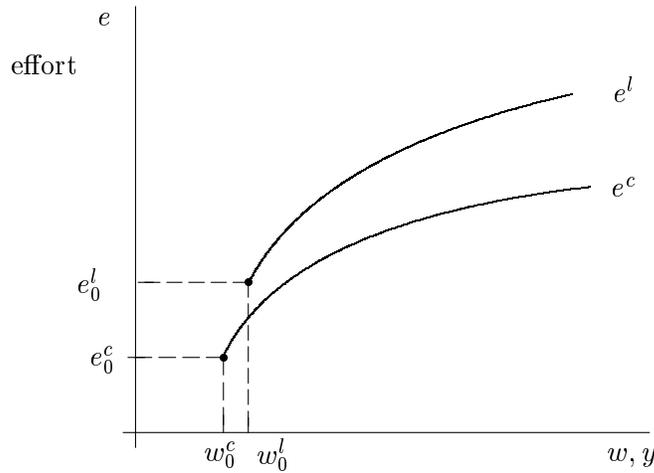

Figure 1: The Superiority of the Employee-run Firm in Eliciting Effort: The shift from the conventional to the employee-run firm provides incentives for peer monitoring and thus changes the typical worker's best response function from e^c to e^l, where greater effort is forthcoming for a given wage and monitoring intensity.

3   WEALTH AND CREDIT IN THE EMPLOYEE-RUN FIRM

While the employee-run firm may dominate the conventional firm in the regulation of work effort, this is not sufficient to ensure its competitive survival, which requires that the employee-run firm be able to recruit members together with their assets in competition with other contractual forms. In this section we show that the probability of firm failure is a decreasing function of the equity/capital ratio (Lemma **??**), the cost to firm members of supply-

ing equity capital falls as their wealth rises (Lemma **??**), the equity/capital ratio increases with the level of member wealth (Theorem **??**), and for these reasons the present value of membership in the employee-run firm is an increasing function of worker wealth (Theorem **??**). Thus when worker wealth is sufficiently high, the employee-run firm's comparative advantage in regulating work effort may render the firm competitively viable.

The following reasoning suggests that the capital costs of the employee-run firm fall when worker wealth increases. An increase in worker wealth reduces the subjective cost of the worker's equity in the firm, both because individuals are decreasingly risk averse (an assumption of our model) and because the share of the worker's assets that are imperfectly diversified (i.e., its equity in the firm) falls. This lower subjective cost of equity raises the gains from participating in the firm by directly increasing the value of participation, and by leading the firm to increase its equity/debt ratio, which in turn lowers its probability of failure, thus allowing the employee-run to finance its debt at lower cost. Thus the costs of both debt and equity fall as worker wealth rises.[23]

A capital market with complete contracts of course does not give us these results, since equity plays no role in such markets. But capital markets are not generally of this type, and lenders must generally devise incentives that induce borrowers to increase the probability of repayment (**?**), including requiring the borrower to share in the equity of a project. Prospective employee-owners face a number of disabilities in this respect. First, since workers receive employment rents (equal to the cost of job loss), they profit from the firm's continued operation even when its expected future profits are negative, whereas creditors prefer to declare bankruptcy in such a situation (**?**). This heightens the incentive incompatibility between creditors and borrowers. Second, it is more costly for outside creditors to provide adequate incentives to a large employee-run organization than to a small group of managers in the conventional firm, since in the employee-run firm these incentives must be extended to at least a majority of members, while in the conventional case a relatively small number of agents must be influenced (**?**). In both cases, a significant degree of worker equity ownership will normally be necessary to secure access to outside credit.

Wealth-poor members of a employee-run firm are unable to attenuate this incentive incompatibility sufficiently by supplying equity to the project. The employee-run firm will thus face a credit and capital market disadvantage. We now demonstrate this insight formally.

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[23]For recent evidence see **?**).

We first show that the probability of firm success p^l increases with the level of equity contributed by the members of the employee-run firm. Let k be worker equity, so the firm borrows an amount $1 - k$. We assume that if the firm cannot repay its loans in full, it is declared bankrupt, its net revenue is dissipated in bankruptcy proceedings, and the salvage value of the firm's capital stock is zero.[24] We assume the employee-run firm has the same exogenous probability γ as the conventional firm of failure for non-financial reasons, and in addition can fail by not meeting its debt obligations. We write

$$\hat{q} = qe^l - y - m + \epsilon,$$

a random variable representing the net revenue of the firm after paying its labor costs y and monitoring costs m, and hence available for servicing its debt and equity obligations. If $r^l(k)(1 - k) < \hat{q} < E\hat{q}$, creditors are paid in full and equity holders are not. But if $\hat{q} < r^l(k)(1 - k)$, by assumption the firm fails. From these considerations is follows that

$$p^l = Pr\left\{\hat{q} \geq \left[\frac{1 + \rho}{p^l} - 1\right](1 - k)\right\},$$

since given p^l, risk-neutral lenders must receive then a nominal interest rate $(1 + \rho)/p^l - 1$ to obtain the risk-free interest rate ρ. Let G be the cumulative distribution of $\hat{q}$. Then assuming that the stochastic term in $\hat{q}$ is independent from the effort level e^l, we have[25]

$$p^l + G\left[\left(\frac{1 + \rho}{p^l} - 1\right)(1 - k)\right] = 1. \tag{6}$$

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[24] Assuming that lenders have some payment level below the full value of the debt that 'triggers' bankruptcy, and that the capital stock is partially salvageable would not alter our conclusions. Similarly, we could assume that worker-owners would be willing to accept less than the wage-surrogate y to save the firm from bankruptcy. This would change the probability of bankruptcy and improve the viability of the employee-run firm, but it would not alter our analysis.

[25] Since equation (??) is somewhat novel, we have calculated some typical cases (calculations available upon request from the authors). Suppose $Pr\{\hat{q} = 0\} = \gamma$, and $\hat{q} \mid \hat{q} \neq 0$ is uniformly distributed on the interval $[0, \beta]$, with expected value $(1 - \gamma)\beta/2$ for some $\beta > 1$. Suppose $\rho = 0.1, \gamma = 0.1$, and we set β so that $\hat{q}$ has net expected return r for various values of r. Then for $r = 10\%$, all equity/capital ratios compatible with (??) and $p^l > 0$ are greater than 0.91; for $r = 34\%$, all equity/capital ratios satisfying (??) and $p^l > 0$ are greater than 0.70; and so on. When $\hat{q}$ is normally distributed, we find similar magnitudes for the lower bound of equity/capital ratios entailing positive probabilities of firm survival. For instance, if $\rho = 10\%$ and the expected return is 20%, when the standard error is $\sigma = 0.06$, $\gamma = 0.04\%$, the corresponding minimum equity/capital ratio is 0.35, and when the standard error is $\sigma = 0.15$, $\gamma = 9\%$, the minimum equity/capital ratio rises to 0.79.

If the employee-run firm is fully internally funded (i.e., $k = 1$), it can fail only for non-financial reasons, so we must have $G(0) = \gamma$. We incorporate this into

Assumption 1 *Let $G(x)$ be the cumulative distribution of the net revenue of the employee-run firm. Then $0 < \gamma = G(0) < 1$; i.e., a fully internally funded employee-run firm has a positive probability of both success and failure. Also $G(x)$ is twice differentiable on the interior of its support, is right continuous at $x = 0$, and the support of the density $g(x) = G'(x)$ is a (possibly infinite) interval containing the origin.*

Lemma 1 *Given Assumption ??, equation (??) has a solution $p^l(k)$ on an open interval $(k_{min}, 1)$, and $p^l(k)$ is continuously differentiable and increasing on this interval.*

The proof of this and other propositions in the paper are given in an Appendix A.

By Lemma **??**, the larger the equity in the employee-run firm, the lower the probability of bankruptcy. But by increasing their equity in the firm, workers increase their exposure to risk, and hence the minimum expected return required to induce them to accept this risk increases. We take workers to be decreasingly risk averse, so the certainty equivalent of a risky lottery increases with increasing wealth. Each worker has wealth W which, we assume, earns the risk-free interest rate ρ. The worker must thus transfer an amount k from the portfolio W to purchase a risky asset: firm membership. To specify the shape of $r(k, p^l, W)$, the expected rate of return on k that renders the worker indifferent between investing in the firm and investing k in an optimally diversified portfolio, let $V(W)$ be the worker's present value of having wealth W.[26]

Summarizing these assumptions and adding formal details, we have

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[26]For simplicity we assume all workers have the same wealth, which is of course unrealistic given the stochastic character of our model. We could relax this assumption by letting $r(k, W)$ be the expected rate of return on k that renders the worker of wealth W indifferent between investing in the firm and investing k in an optimally diversified portfolio. For any interest rate $\tilde{r}$, a member of wealth W is willing to supply capital $k(\tilde{r}, W)$ such that $\tilde{r} = r(k(\tilde{r}), W)$. If $\phi(W)$ is the density of wealth, the maximum equity from members forthcoming at interest rate $\tilde{r}$ is

$$\tilde{k}(\tilde{r}) = \int k(\tilde{r}, W)\phi(W)dw.$$

Since $\tilde{k}(\tilde{r})$ is an increasing function over the relevant range, we can use its inverse $\tilde{r}(\tilde{k})$ in the place of $r(k, W)$ in the argument below. Our analysis also trivially extends to showing that an increase in employee wealth lowers the $\tilde{r}(\tilde{k})$ schedule, so all our results extend

Assumption 2 *Each member is decreasingly risk averse and has wealth W that earns the risk-free interest rate ρ. The member's present value of having wealth W is $V(W)$, which is increasing, concave and twice differentiable in W.*[27]

Lemma 2 *Consider an investment of size k in a risky asset (firm membership) that has a probability of success p^l. Let $r(k, p^l, W)$ be the rate of return on this investment that renders an agent with wealth W just willing to move k from the risk-free portfolio to the risky asset. Then Assumption ?? implies $\partial r / \partial W < 0$; i.e., an increase in wealth leads the agent to accept a lower interest rate for any given level of k and p^l.*

Proof: The return $r(k, p^l, W)$ must satisfy

$$V[(\rho+1)W] = pV[(\rho+1)W + (r(k, p^l, W) - \rho)k] + (1 - p^l)V[(\rho+1)(W - k)]. \tag{7}$$

In Theorem A?? in Appendix A we show that (??) implies $\partial r / \partial W < 0$. The intuition motivating the proof is given in the second paragraph of this section. $\square$

For expositional purposes, we would like to exclude corner solutions in which the employee-run firm is fully internally funded ($k = 1$) or the members' wealth is wholly invested in the firm's equity ($k = W$). To exclude these cases we make the plausible assumption $1 > W$,[28]), and $V(W) \to -\infty$ as $W \to 0$. For given the latter assumption, $k \to W$, $r \to \infty$, so $p^l \leq 1 - \gamma < 1$ implies that the final term in (??) approaches $-\infty$.[29] This motivates

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to the case of heterogeneous wealth. With these changes, the propositions in Section ?? relating to W would have to be replaced by propositions relating to the distribution of W among workers.

[27] The assumption that the worker's portfolio is risk-free could be weakened by adjusting ρ upward and letting $V(\cdot)$ be the present value of a stochastic stream of portfolio returns. We assume the risk-free case for simplicity.

[28] For a rough sense of the relevant wealth constraints, consider that in 1988 the average wealth (including car and home) of the least wealthy 80% of U.S. families was about $64,000 (half of which was car and home). The capital stock in the U.S. economy per employee (roughly our **k** times the number of hours of labor) was about $95,000 , and the number of employed workers per family was about 1.3; the statistics are from ?) and the ?). Thus total net worth of a typical worker is about half the value of the capital stock they typically work with.

[29] We could of course ensure an interior solution with a less extreme condition on $W[0]$. But since our analysis goes through even when the corner solutions are equilibria, it would serve no purpose to choose a weaker condition.

Assumption 3 *Member wealth is less than the firm's per worker capital stock; i.e., $W < 1$. The utility of zero wealth is $-\infty$; i.e., $V(W) \to -\infty$ as $W \to 0$.*

Note that even wealth levels somewhat greater than $\mathbf{k} = 1$ will preclude full ownership by team members as the implied concentration of assets would make membership an unattractive investment. However the assumption that $W < \mathbf{k}$ is not unrealistic. We will also need the following somewhat technical

Assumption 4 *For a risky asset with expected rate of return r, let k be such that the agent is indifferent between holding a risk-free portfolio of size W and a portfolio with k in the risky asset and $W - k$ in risk-free assets. Then $\partial k / \partial W > 0$.*

Note that this assumption is true if the agent has constant relative risk aversion,[30] but is clearly true under considerably weaker conditions. It may follow from decreasing risk aversion, but we have not been able to establish this fact. At any rate, since the condition of Assumption **??** could reasonably serve as the very *definition* of decreasing risk aversion, we consider the assumption quite plausible.

To determine the optimal level of equity k^* in the firm, we note that the probability of job loss is $1 - p^l + p^l f^l(e^l)$ where $1 - p^l$ is the probability of bankruptcy and $f^l(e^l)$ is the probability of dismissal. Then (**??**) can be written as

$$v^l = \frac{u(y, e^l) - \rho z}{\rho + 1 - p^l + p^l f^l(e^l)} + z \qquad (??a)$$

where y is now given by

$$y = q(e^l) - m - \frac{(1 + \rho - p^l)(1 - k)}{p^l} - r(k, p^l, W)k. \qquad (??b)$$

Analogous to the case of the conventional firm, the worker's best response function $e^l(y, p^l)$ is given by the effort level that maximizes (**??**a) subject to (**??**b), or[31]

$$\frac{dv^l}{de^l} = \frac{u_e - (v^l - z)p^l f_e^l}{\rho + 1 - p^l + p^l f^l} = 0, \qquad (8)$$

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[30]To see this, note that there are only three forms V(W) can have (up to an affine transformation): $V(W) = W^\alpha, 1 > \alpha > 0$; $V(W) = -W^\alpha, \alpha < 0$; and $V(W) = log(W)$. In each of these cases, algebraic manipulation of Equation **??** shows that k and W appear only in the form k/W.

[31]Recall that teams are sufficiently large that no individual team member can increase the probability of firm success by individually increasing effort, so p^l is treated as a parameter by each team member.

the interpretation of which is entirely analogous to that of equation (**??**). Just as in the case of equation (**??**), equation (**??**) defines a best response function $e^l = e^l(y, p^l)$, where p^l is included as an argument because it is variable and endogenously determined by management decisions in the case of the employee-run firm (for the conventional firm $p^c = 1 - \gamma$ is constant).

Managers of the employee-run firm know members' best response functions and are instructed to choose k to maximize (**??**a) subject to $e^l = e^l(y, p^l(k))$ and with y given by (**??**b).[32] After some algebraic manipulation the first order condition for optimal equity k can be written as

$$\frac{dv^l}{dk} = \frac{u_y \frac{dy}{dk} + (v^l - z)(1 - f^l(e^l))p_k^l}{\rho + 1 - p^l + p^l f^l(e^l)} = 0 \qquad (9)$$

with

$$\frac{dy}{dk} = \frac{q' e_p^l + \frac{1}{p^{s2}}\left[(\rho + 1 - p^l)p^l p_k^l + (1 + \rho)(1 - k)\right] - \frac{d\mathcal{D}}{dk}}{1 - q' e_y^l} \qquad (10)$$

where $\mathcal{D}(k, p^l) = r(k, p^l, W)k$ is the firm's cost of servicing equity and $d\mathcal{D}/dk = kr_k(k, p^l) + r(k, p^l, W)$. Thus

$$\frac{d\mathcal{D}}{dk} = \mathcal{D}_k + kr_p(k, p^l)p_k^l.$$

We interpret equation (**??**) as follows. An increase of Δk in k leads to a decrease in the probability of bankruptcy $p_k^l \Delta k$, which reduces the expected cost of job loss through bankruptcy by $(v^l - z)(1 - f^l(e^l))p_k^l \Delta k$, and increases the flow of utility from income by $u_y \frac{dy}{dk} \Delta k$. This then is all discounted to the present by $\rho + 1 - p^l + p^l f^l(e^l)$. At an optimum we must then have $dy/dk < 0$. The denominator of (**??**) for dy/dk is a 'multiplier' term that must be positive at a maximum, or else promising members a larger share Δy would be feasible, since it would lead to an increase in output $q' e_y^l \Delta y$ greater than Δy. The first term in the numerator of (**??**) is the direct effect on y of an increase in job security on effort and hence output, the second term is the reduction in debt service caused by a lower default rate, and the final term is the increase in the cost to members of supplying equity.

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[32]By construction, y represents the total expected flow of gains to firm membership, since the equity return $r(k, p^l, W)k$ is set to render the member indifferent between holding the risky asset (firm membership) and a risk-free equivalent. Since z does not include returns accruing to the worker independent of tenure with the firm, it is a reasonable approximation to treat z as independent of worker wealth, and hence the same whether the worker is dismissed or the firm becomes bankrupt. In a more complete treatment, z would differ in the two cases (for instance, because a worker with more wealth may be able to sustain a more lengthy or costly job search).

The determination of k is depicted in Figure **??**, where the 'Marginal Benefit of Equity' schedule refers to all the terms in equation (**??**) except $\frac{d\mathcal{D}}{dk}/(1 - q'e_y^l)$, after substituting in the expression for dy/dk from equation (**??**), and the 'Marginal Cost of Equity' schedule represents $\frac{d\mathcal{D}}{dk}/(1 - q'e_y^l)$. The optimal level of worker equity k^* occurs where the two schedules intersect.
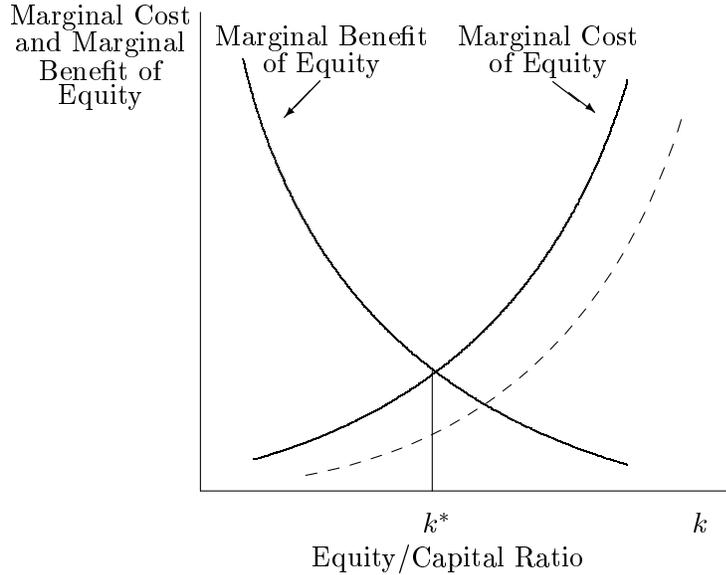

Figure 2: The Internal Finance of the Employee-run Firm: An increase in wealth shifts the Marginal Cost of Equity schedule rightward, leading to a higher equilibrium equity/capital ratio k^* and a lower marginal equilibrium marginal cost of capital.

It is clear from Figure **??** that $dk^*/dW > 0$, since a rise in W shifts down the marginal cost of equity schedule $d\mathcal{D}/dk$ without affecting the marginal benefits of equity schedule, so its intersection with the benefits of equity schedule shifts to the right. Formally we prove

Theorem 1 *The optimal equity level k^* is an increasing function of worker wealth; i.e., $dk^*/dW > 0$.*

To prove Theorem **??**, note that $v_k^l = 0$ implies $v_{kk}^l k_W + v_{kW}^l = 0$, and since $v_{kk}^l < 0$ by the second order condition, we need only show that $v_{kW}^l > 0$, which says that the marginal contribution of equity to the value of member-

ship is greater, the greater the wealth of members. But (**??**) implies

$$v_{kW}^l = -u_y \frac{\frac{\partial}{\partial W}\frac{d\mathcal{D}}{dk}}{(\rho + 1 - p^l + p^l f^l)(1 - q'e_y^l)},$$

so to complete the proof we must show that $\frac{\partial}{\partial W}f^l\frac{d\mathcal{D}}{dk} < 0$, which says that the marginal cost of acquiring members' equity varies inversely with their wealth. This intuitively reasonable property of the model is proved in *Theorem A*??? in Appendix A.[33] $\square$

How does worker wealth W affect the present value v^l of being a member of the employee-run firm? We have already seen that unless W is sufficiently large, the employee-run firm is not viable at all, since no debt contract is acceptable to lenders unless the equity/capital ratio is sufficiently high. More generally we have

Theorem 2 *The present value of membership in the employee-run firm is an increasing function of member wealth; i.e., $dv^l/dW > 0$.*

Proof: Member wealth W affects the firm's equilibrium only via its effect on the schedule $r(k, p^l, W)$, which appears only in the binding constraint Equation **??**b. Lemma **??** shows that an increase in W shifts the schedule $r(k, p^l, W)$ down, and hence entails an increase in y, which leads to an increase in v^l. While k is no longer necessarily optimal, the new optimal v^l is necessarily greater than the old. This proves that $dv^l/dW > 0$. $\square$

This relationship between member wealth W and the present value of being a member of the employee-run firm v^l is depicted in Figure **??**. Note that at some level of worker wealth, labeled W^+ in Figure **??**, workers can borrow and supply equity on the terms equivalent to the owners of conventional firms. Thus the difference $v^l(W^+) - v^c$ is a measure of the superior effort regulation capacities of the employee-run firm. Let W^* be the worker wealth level such that $v^c = v^l(W^*)$, and consider W^0, some level of worker wealth such that $W^* < W^0 < W^+$. We can decompose the advantage of the employee-run firm in worker expected utility terms, $v^l(W^0) - v^c$, into its effort regulation superiority, $v^l(W^+) - v^c$ minus its credit market disability, $v^l(W^+) - v^l(W^0)$, as shown in Figure **??**.

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[33]Note that we are assuming for simplicity that u_y does not depend on W. If in fact the utility of income fell sufficiently fast, the level of worker wealth that overcomes the employee-run firm's capital market disadvantage may be so high that workers prefer leisure to work, rendering neither the conventional nor the employee-run firm viable.
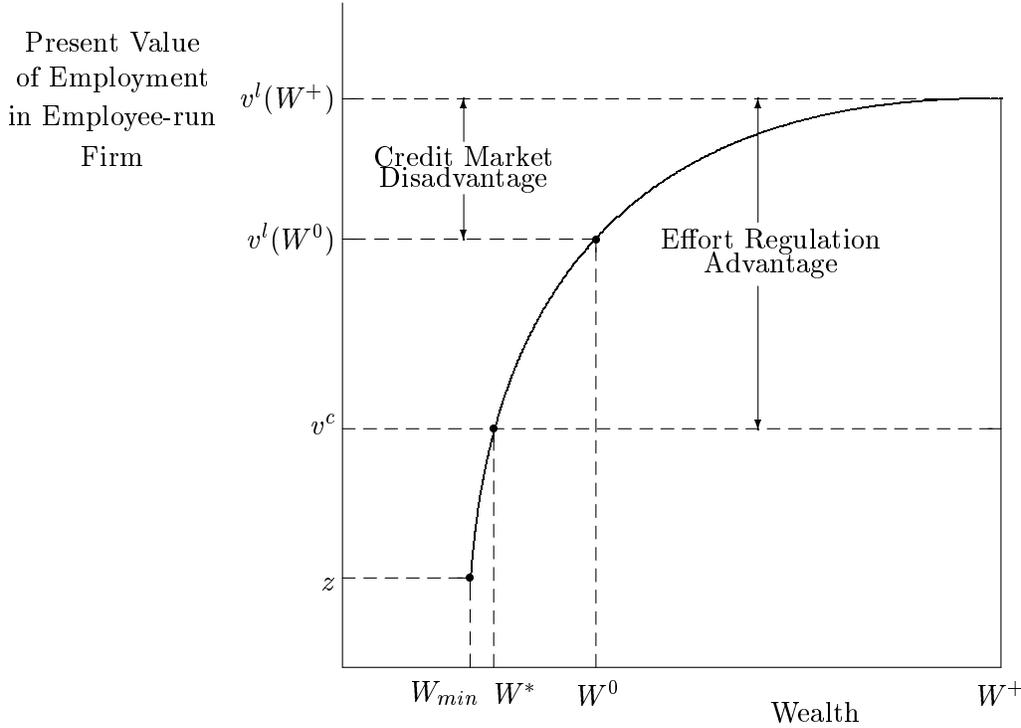
Figure 3: Worker Wealth and Viability of the Employee-run Firm

4   WEALTH EFFECTS ON THE DISTRIBUTION OF CONTROL RIGHTS

The fact that the cost of capital to the employee-run firm decreases as worker wealth increases implies that the fraction of workers in employee-run firms is an increasing function of worker wealth. Our reasoning is as follows. We can expect the various sectors of the economy to have distinct levels of capital per worker, and to experience differential gains in effort regulation associated with employee ownership.[34] At low levels of worker wealth, the financial burden of the employee-run firm is likely to outweigh the benefits in the regulation of labor in all but the most auspicious environments, namely those with low capital requirements and production processes for which the

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[34]A more general treatment would take account of the degree of asset specificity of the capital stock, with employee-run firms being disfavored less in sectors with little firm-specific assets.

employee-run firm's labor regulation advantages are especially great. But as worker wealth increases, sectors of the economy that previously had excessive capital intensity or insufficient gains in effort regulation from being run by employees, will come to exhibit greater returns to employee-ownership than to conventional ownership. In the limit, with worker wealth such that workers become risk-neutral, employee-run firms will be superior in all sectors in which employee-ownership dominates conventional organization in the regulation of effort.

We assume that both conventional and employee-run firms hire from the pool of dismissed workers, and newly dismissed workers return to this pool. A firm can be transformed from one type to the other should one form of governance offer superior returns to the workers in it: if the present value of the job under employee control exceeds that under conventional control ($v^l > v^c$), the workers will purchase the firm, financing the purchase in the manner indicated in the previous section. Conversely, members of an employee-run firm in a sector for which $v^l < v^c$ will sell the firm to a conventional owner, liquidating their shares and returning their assets to their (presumed) risk-free portfolio. The employee-run firm is said to be *viable* if there exists at least one sector (indexed by i) for which $v_i^l > v_i^c$, and which is therefore composed of employee-run firms. We define $\underline{W}$ as the minimum level of wealth for which the employee-run firm is viable.

To explore the effect of the level of worker wealth on $\mathcal{F}$, the fraction of the labor force in employee-run firms, suppose we can define 'sectors' of the economy sufficiently narrowly that for levels of wealth sufficient to make the employee-run firms viable, there exists at least one sector j for which $v_j^l(W) = v_j^c$, and in which, for this reason, both types of firms coexist.[35] Because the present value of membership is an increasing function of worker wealth (Theorem **??**), an increase in worker wealth must prompt workers in this sector now employed in conventional firms to convert to employee management, with the appropriate financial arrangements. Thus for $\underline{W} < W < W^+$, the fraction of workers $\mathcal{F}$ in employee-run firms will vary directly with the level of worker wealth; i.e., $\mathcal{F} = \phi(W)$ with $\phi' > 0$.

We now show that the equilibrium level of worker wealth is an increasing function of the fraction of workers in employee-run firms. For the intuition behind this result, consider a worker who is just indifferent to holding a position in a conventional and a employee-run firm (that is, $v^l = v^c$ for this

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[35]We use the fiction of arbitrarily narrowly defined sectors simply as an expedient justifying our (implicit) assumption that the function $\phi(W)$ below is continuous. It plays no substantive role in the argument, which could readily be rephrased in terms of discrete changes, or of functions with a finite number of step discontinuities.

worker). We have seen that the worker must be offered a risk premium to be induced to hold equity in the employee-run firm. Thus when $v^l = v^c$, expected income will be higher with the employee-run firm, and hence even with a constant savings ratio, the worker's expected future wealth will be higher as well. But assuming decreasing risk aversion, increased risk exposure will induce the worker save more. To see this, note that by increasing wealth (at the expense of a lower flow of consumption expenditure), the worker moves to a less risk-sensitive region of the utility function. In addition a higher savings rate, by increasing the worker's non-firm-specific wealth, offers liquidity benefits: the worker can reduce the probability of being credit-rationed on the down-side of economic fluctuations.

To model this process, suppose a typical worker in the employee-run firm has initial wealth W, invests k in the firm, receiving an expected return r, has labor income y, and has consumption expenditure c.[36] The worker's next-period wealth W_1 is then given by

$$W_1 = (1 + \rho)(W - c) + y + \mu, \tag{11}$$

where μ is a random variable representing the worker's expected residual claim on the profits of the firm after paying its fixed obligations to creditors and workers, plus the contingency of job loss because of termination and capital loss because of firm bankruptcy.

We may write the worker's present value of utility (after having chosen an optimal level of effort) as $U(c, W_1)$. We assume U is increasing and concave in c and W_1, U_W is convex (i.e., the worker is decreasingly risk averse), and U is additively separable in c and W_1 (i.e., the present value of utility is the discounted sum of consumption utilities in each time period). Since W_1 is a random variable, we can characterize the worker's problem as choosing c to maximize $EU(c, W_1)$ subject to (**??**), where the expectation is taken with respect to μ. We also assume that optimal consumption is neither equal to zero nor equal to $W - k$, so the worker's choice of consumption is an interior solution to the problem

$$\max_c EU(c, W_1)$$

subject to (**??**). Let σ be a measure of the risk associated with the random

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[36]We may justify a 'representative worker' approach by considering workers as infinitely lived, moving in and out of employment with a rate such that a worker's current wealth W is unrelated to whether employed or unemployed, as well as length of current employment. It would be desirable to have a more complete analysis in which wealth is a function of employment history.

return μ.[37] If $c^*(W)$ is the solution to this problem, we must show that $dc^*/d\sigma < 0$, so the rate of saving increases with riskiness.

The first order condition for c is

$$U_c(c^*) = (1 + \rho)EU_{W_1}(W_1). \tag{12}$$

To show that $dc^*/d\sigma < 0$, it is sufficient to show that the right hand side of (??) is convex in W, for then $dEU_W/d\sigma > 0$ (see the previous footnote). It follows that an increase in risk implies an increase in the right hand side of (??), which implies an increase in $U_c(c^*)$, and since U is concave, this implies a decrease in c^*. We write $V(W) = max_c EU(c, W_1)$ where W_1 is given by (??). We show in Theorem A?? of Appendix A that V is increasing, concave, and dV/dW is convex. But

$$\frac{dV}{dW} = E\left\{U_c(c^*)\frac{dc^*}{dW} + U_W(\rho + 1)\left(1 - \frac{dc^*}{dW}\right)\right\} = (\rho + 1)EU_W,$$

where we have used (??) to simplify the middle expression. Since dV/dW is convex in W, so is $(\rho + 1)EU_W$, which proves our assertion.

It follows that an increase in the fraction $\mathcal{F}$ of workers in employee-run firms will increase expected worker wealth. Thus we can define the wealth level of workers as an increasing function of the fraction of employee-run firms:

$$W = \psi(\mathcal{F}), \qquad \psi' > 0. \tag{13}$$

5 The Joint Determination of the Fraction of Democratic Firms and the Distribution of Wealth

Our analysis implies that the equilibrium values of $\mathcal{F}$ and W are jointly determined by two functions, $\mathcal{F} = \phi(W)$ and $W = \psi(\mathcal{F})$. We illustrate both function in Figure ??. Both functions are increasing, since as we have seen, greater worker wealth entails a larger fraction of employee-run firms, and a larger fraction of employee-run firms implies greater worker wealth.

In Figure ?? we interpret both functions as stationarity conditions for their respective variables. Thus

$$\frac{d\mathcal{F}}{dt} = \alpha(\phi(W) - \mathcal{F}) \qquad\qquad \alpha(0) = 0, \alpha' > 0. \tag{14}$$

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[37]We say a random variable μ_1 is riskier than μ_0 if μ_1 can be written as the sum of μ_0 and a random variable ϵ such that $E(\epsilon \mid \mu_0) = 0$. This is equivalent to every risk-averse agent preferring μ_0 to μ_1. Thus the expected value of a concave function of a risky random variable decreases when risk increases, while a convex function of a risky random variable increases with risk.

and

$$\frac{dW}{dt} = \beta(\psi(\mathcal{F}) - W) \qquad\qquad \beta(0) = 0, \beta' > 0. \qquad\qquad (15)$$

The arrows in Figure **??** show the movement of the variables out of equilibrium. The reader may confirm that the equilibrium $(W^*, \mathcal{F}^*)$ is stable. The equilibrium distribution of wealth and of workers among firms $(W^*, \mathcal{F}^*)$ is that for which the savings behavior generated by $\mathcal{F}^*$ supports the wealth level W^* which in turn supports $\mathcal{F}^*$.
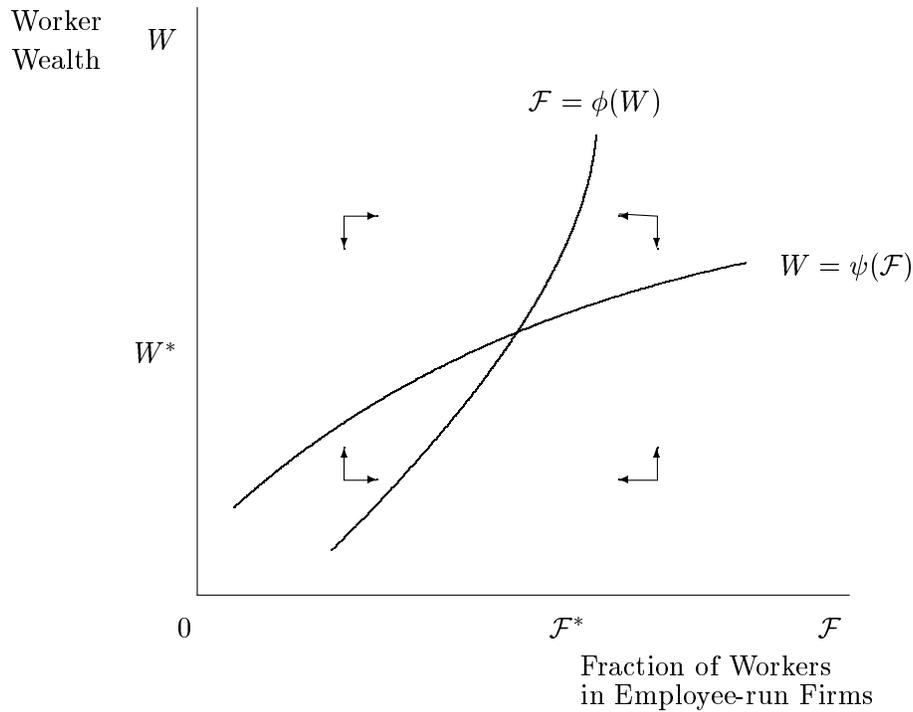

Figure 4: Mutual Determination of Worker Wealth and the Fraction of Workers in Employee-run Firms

It follows that the level of worker wealth and the fraction of employee-run firms can both be increased by shifting one or both of the functional relationships. The $\phi(W)$ schedule can be shifted out through a policy of improving credit availability to workers, insuring worker firm-specific assets, or subsidizing investment in employee-run firms, for example by treating worker membership shares as equivalent for tax treatment to investment retirement accounts.[38]   The $\psi(\mathcal{F})$ schedule can be shifted up by a direct

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[38]If these programs are financed by a tax on worker wealth, $\phi(W)$ can be shifted without

wealth redistribution toward workers or by subsidizing worker saving.

   Given that both schedules are upward rising, the possibility of multiple equilibria cannot be excluded. We illustrate this possibility in Figure **??**, in which we assume, plausibly we think, but without evidence, that both the ϕ and ψ functions are positive and concave in their arguments. Here there are two stable equilibria, a low-level equilibrium where $\mathcal{F} = 0$ and a high level equilibrium at $(\mathcal{F}^*, W^*)$. There is also an unstable equilibrium at $(\mathcal{F}_*, W_*)$. Given the out-of-equilibrium dynamics determined by (**??**) and (**??**), should the economy be at the low level equilibrium, the high level equilibrium could be attained by redistributing wealth to some point beyond W_*, or any policy inducing an increase in the fraction of workers in employee-run firms beyond $\mathcal{F}_*$.
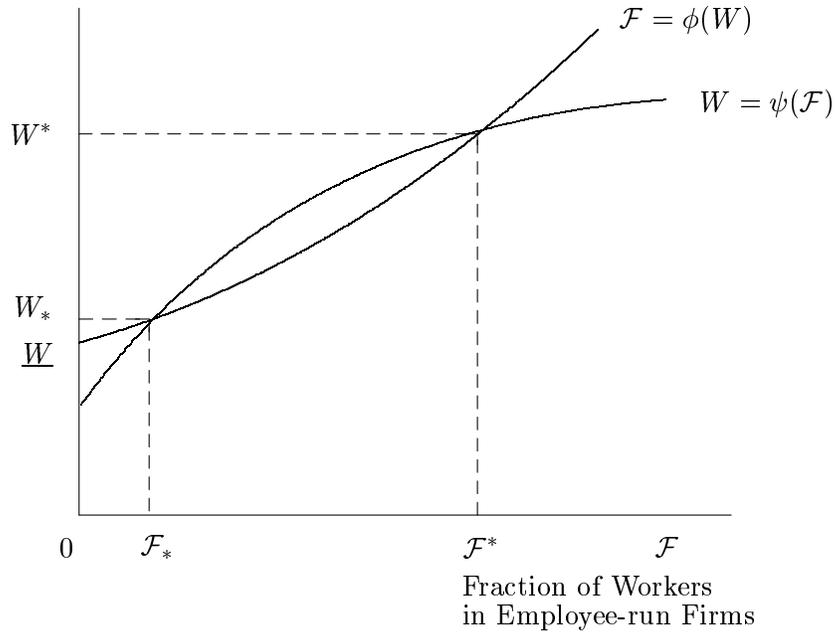
Worker
Wealth

Figure 5: **Multiple Equilibria in the Incidence of Employee-run Firms**

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shifting the $\psi(\mathcal{F})$ schedule.

6 CONCLUSION

It is commonly asserted that the assignment of control rights observed in market economies is efficient. For instance, writing about worker controlled firms, **?**):473 assert

> The fact that this system seldom arises out of voluntary arrangements among individuals strongly suggests that codetermination or industrial democracy is less efficient than the alternatives which grow up and survive in a competitive environment.

But we have shown that the level of worker wealth and the incidence of employee-run firms are jointly determined, from which it follows that an observed distribution of workers among types of firms does not support inferences about the efficiency or even competitive viability of alternative assignments of control and residual claimancy rights. A move from the equilibrium $(0, \underline{W})$ to $(\mathcal{F}^*, W^*)$ could be Pareto-improving and capable of being implemented by a one time intervention without continuing subsidy, even though such a move might not arise spontaneously. Of course even if multiple equilibria are precluded, there is no mechanism ensuring the efficiency of the competitive distribution of workers among firms if workers' wealth is less than W^+. We can only say that for each level of wealth W the resulting distribution of workers $\phi(W)$ is locally constrained-Pareto optimal: given the informational constraints defining the problem, Pareto-superior alternatives do not exist in the neighborhood of $\phi(W)$ (recall that the variation in $\mathcal{F}$ takes place simply through workers buying out previous nonworker owners).

Our analysis also implies that under the appropriate conditions economic democracy and equality may be complementary objectives in the sense that policies that displace a (stable) equilibrium towards a greater level of wealth for the less wealthy will also support a larger fraction of workers in employee-run firms, and conversely.

While these conclusions offer a greater scope for societal intervention in the pursuit of both employee-run and egalitarian objectives (should these values be widespread enough to warrant action) than suggested by the traditional equity-efficiency trade-off, our analysis does not allow any conclusions about the related question of the efficiency costs of various distributions of wealth and allocation of workers among types of firms.

In particular, our model does not address the important question of long term productivity growth and the possible innovation-dampening effects of the dispersal of control rights implied by the employee-run firm. Nor does it

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permit an exploration of the insurance and other policy measures that might attenuate these effects (though the technological dynamism associated with dispersed ownership in the agricultural sectors of many economies suggests that such policies might be developed).

7 Appendix A

Proof of Lemma **??**: The implicit function theorem, together with the smoothness condition in Assumption **??**, ensure that $p^l(k)$ is continuously differentiable where it exists. Suppose $p^l(k)$ satisfying (**??**) exists for some $k_1 < 1$. Then $p^l(k)$ exists for all k in the interval $(k_1, 1)$. To see this, let us define $F(p^l, \alpha) = G[(\frac{1+\rho}{p^l} - 1)\alpha]$, so (**??**) becomes $F(p^l, 1 - k) = 1 - p^l$. Since $F(p^l(k_1), 1 - k_1) = 1 - p^l(k_1)$ and $F_\alpha > 0$, for $k < k_1$ we have $F(p^l(k_1), 1 - k) < 1 - p^l(k_1)$. But by Assumption **??**, when $p^l = 1$ we have $F(p^l, 1 - k) \geq \gamma > 1 - p^l$, and since F is continuous, by the mean value theorem there is a p^l such that $F(p^l, 1 - k) = 1 - p^l$; i.e., $p^l(k)$ satisfying (**??**) exists. To show that $p^l(k)$ is defined on an interval $(k_{min}, 1)$, we need only show that $p^l(k)$ exists for some $k < 1$. Fix any p^l such that $1 - G(1) < p^l < 1 - G(0)$. Note that such a p^l exists, since the support of $g(x)$ includes the origin, ensuring that $G(1) > G(0)$. By the assumption that $G(x)$ is right continuous at the origin, for sufficiently small $\alpha > 0$ we have $F(p^l, \alpha) < 1 - p^l$. But for $\alpha = 1$ the inequality is reversed. Thus by the mean value theorem $p^l = p^l(\alpha)$ for some $\alpha < 1$. This proves that $p^l(k)$ is defined on an interval. To see that $p^l(k)$ is an increasing function of k on the interval $(k_{min}, 1)$, we differentiate (**??**), getting

$$\frac{dp^l}{dk} = \frac{(1 + \rho - p^l)g}{p^l - (1 + \rho)(1 - k)g/p^l}. \tag{16}$$

When $k = 1$, (**??**) shows that $p^l = 1 - G(0) > 0$, so (**??**) and the fact that p^l is continuously differentiable on an interval implies the denominator on the right hand side of equation (**??**) cannot vanish. Moreover the numerator on the right hand side of (**??**) is strictly positive on $(k_{min}, 1)$, so $p^l(k)$ cannot change sign on this interval and hence is increasing there. □

Theorem A1 *An agent with wealth $W > 0$ invests an amount $k > 0$ in a project with a probability $0 < p < 1$ of being successful. The agent earns risk-free interest rate ρ on $(W - k)$, plus interest rate r on the amount k invested if the project is successful, where the probability of success is p. If r is just sufficient to induce the agent to undertake the investment, and if the agent is decreasingly risk averse and satisfies Assumption **??**, then $\partial r/\partial W < 0$.*

Proof: From equation (**??**) with $\mathcal{E} = (r(k, p, W) - \rho)k$, we have

$$V[(\rho + 1)W] = pV[(\rho + 1)W + \mathcal{E}] + (1 - p)V[(\rho + 1)(W - k)]. \tag{17}$$

Define $u_1(x) = V[(\rho+1)xW]/V[(1+\rho)W]$, $s = (r-\rho)/(1+\rho)$, and $\nu = k/W$. Then (**??**) can be written

$$1 = pu_1(1 + s\nu) + (1 - p)u_1(1 - \nu), \tag{18}$$

and we must show that $s(\nu)$ is increasing in ν. Let

$$m_1(\nu) = \frac{1 - u_1(1 - \nu)}{\nu}, \qquad m_2(\nu) = \frac{u(1 + \nu) - 1}{\nu}.$$

Since $u_1(\cdot)$ is strictly concave, the mean value theorem shows that $m_1(\nu)$ is an increasing function of ν, $m_2(\nu)$ is a decreasing function of ν, and $m_1(\nu_1) < m_2(\nu_2)$ for $0 < \nu_1, \nu_2 < 1$. But (**??**) implies

$$s(\nu) = \frac{(1 - p)m_1(\nu)}{pm_2(s\nu)}$$

Thus

$$s'(\nu) = \frac{1 - p}{p} \frac{m_2(s\nu)m_1'(\nu) - \frac{d(s\nu)}{d\nu}m_1(\nu)m_2'(s\nu)}{[m_2(s\nu)]^2}.$$

We need only show that $d(s\nu)/d\nu > 0$. But differentiating (**??**), we get

$$\frac{d(s\nu)}{d\nu} = (1 - p)u_1'(1 - \nu)/pu_1'(1 + s\nu) > 0.$$

This proves that $ds/d\nu > 0$, which shows that $\partial r/\partial W < 0$. This proves Theorem **??**. *Box*

Theorem A2 *Under the same assumptions as Theorem* **??** *and Section* **??**, $d\mathcal{D}/dk$ *is a decreasing function of* W.

Proof: We have

$$\frac{\partial}{\partial W}\frac{d\mathcal{D}}{dk} = \frac{\partial}{\partial W}\mathcal{D}_k + kp_k r_p(k, p, W). \tag{19}$$

To show that $\mathcal{D}_k$ is a decreasing function of W, it suffices to show that $\mathcal{E}_{kW} < 0$, where $\mathcal{E} = (r(k, p, W) - \rho)k$. Writing $u(x) = V((1 + \rho)x)$, (**??**) becomes

$$u(W) = pu(W + \mathcal{E}) + (1 - p)u(W - k). \tag{20}$$

Differentiating with respect to k, we get

$$\frac{u'(W - k)}{u'(w + \mathcal{E})} = p\mathcal{E}_k/(1 - p).$$

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Now we differentiate this identity with respect to W, getting

$$\frac{u''(W-k) - u''(w+\mathcal{E})\mathcal{E}_W u'(W-k)/u'(W+\mathcal{E})}{u'(w+\mathcal{E})} = p\mathcal{E}_{kW}/(1-p),$$

which it negative if $\mathcal{E}_W < 0$. But Lemma ?? implies $\mathcal{E}_W = r_W(k, p, W)k < 0$.

It remains to show that $\partial^2 r(k, p, W)/\partial p \partial W < 0$. Differentiating equation (??) totally with respect to p, we get

$$\frac{\partial r}{\partial p} = -\frac{V[W(1+\rho) + (r-\rho)k] - V[(1+\rho)(W-k)]}{pkV'[W(1+\rho) + (r-\rho)k]} < 0.$$

Differentiating this expression with respect to W, we get

$$\frac{\partial^2 r}{\partial p \partial W} = \frac{V'[W^+] - V'[W^-] - kpr_p V''[W^+]}{kpV'[W^+]}.$$

where $W^+ = W(1+\rho) + (r-\rho)k > W^- = (1+\rho)(W-k)$. The second term in the numerator of this expression is greater that the first, since $V'' < 0$, and the third term is positive since r_p is negative. The denominator is positive, so the whole expression is negative. This shows that the derivative of the right hand side of equation (??) with respect to W is negative, which proves Theorem ??. $\square$

Theorem A3 *Let $V(W) = \max_c EU(c, W_1)$ where W_1 is given by equation (??). Then $V(W)$ is increasing and concave, and dV/dW is convex.*[39]

Proof: Write $\rho' = 1 + \rho$. We have shown that $V_W = \rho' EU_W > 0$, so V is increasing. Differentiating again, we get

$$V_{WW} = \rho'^2(1 - c_W)EU_{WW} < 0,$$

so V is concave. Also

$$V_{WWW} = \rho'^2 \left\{ -\frac{d^2 c}{dW^2} EU_{WW} + \left(1 - \frac{dc}{dW}\right)^2 \rho' U_{WWW} \right\}. \qquad (21)$$

We will show that $0 < dc/dW < 1$ and $d^2 c/dW^2 < 0$, which proves the theorem. Differentiating (??) with respect to W, we get

$$\frac{dc}{dW} = \frac{\rho'^2 EU_{WW}}{U_{cc} + \rho'^2 EU_{WW}}. \qquad (22)$$

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[39]Models of this type have been investigated by ?) and ?).

Since U_{cc} and EU_{WW} are negative, $0 < dc/dW < 1$. Differentiating (??) again with respect to W, we get

$$E[U_{cc} + \rho'^2 U_{WW}]\frac{d^2c}{dW^2} + EU_{ccc}\left(\frac{dc}{dW}\right)^2 = \rho'^3 \left(1 - \frac{dc}{dW}\right)^2 EU_{WWW}.$$

If we substitute this expression in (??), we get

$$V_{WWW} = EU_{ccc}\left(\frac{dc}{dW}\right)^3 > 0. \tag{23}$$

This proves the theorem. $\square$

8   APPENDIX B

We have assumed all workers have equal wealth in our analysis. This assumption can be dropped with some minor changes in the analysis. Assuming the employee-run firm does not know individual worker wealth, it is clear that the firm will set a minimum equity requirement for membership, and may allow workers to acquire equity above this cutoff.[40]

We have shown that the fraction $\mathcal{F}$ of workers in employee-run firms contributes directly to the increase in W. However it is clear than in this case, since participation in employee-run firms increases the risk exposure as well as the expected return to workers, the dispersion as well as the mean of W may increase together. We thus close this section by arguing that under plausible conditions our model remains valid in this more general case.

Suppose wealth W among workers is distributed as $W + x$, where x is a random variable of zero mean and density $g(x, \sigma)$, σ being a measure of the riskiness of the distribution underlying x. Let $\underline{W}$ be the minimum wealth required to be a member of an employee-run firm, and suppose when mean wealth W increases, the dispersion of wealth changes according to $\sigma = \sigma(W)$, where $\sigma_W > 0$. We will show that $d\mathcal{F}/dW > 0$, so our model continues to

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[40]The proper level of this minimum is an interesting an largely unexplored question. If the minimum is too low, the equity/capital ratio will be suboptimal. If the minimum is too high, low wealth workers will put up the required capital as long as the employment rents they receive exceed the subjective costs of a high level of risk exposure. But this reduces the cost of job loss to such workers, and hence lowers their optimal effort. On the other hand, in the long run low wealth workers will be dismissed at a higher rate than high wealth workers, for this very reason. The optimum minimum equity requirement must balance these conflicting factors. It is also clearly Pareto-improving to allow workers to purchase equity above this minimum level, unless the resulting heterogeneity of financial interests in the firm among workers increases the costs of employee-management.

hold in this more general case. We have $\mathcal{F} = 1 - G(\underline{W} - W, \sigma(W))$, where G is the cumulative distribution of the dispersion x of wealth W around its mean W. Then

$$\frac{d\mathcal{F}}{dW} = g(\underline{W} - W, \sigma) + \sigma_W \int_{\underline{W}-W}^{\infty} g_\sigma(x, \sigma(W))dx. \qquad (24)$$

Specifically, suppose x is normally distributed with standard deviation σ, so $g(x, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}}$. Then

$$g_\sigma(x, \sigma) = \frac{x^2 - \sigma^2}{\sqrt{2\pi}\sigma^4} e^{-\frac{x^2}{2\sigma^2}}$$

and an integration by parts shows that

$$\int_{\underline{W}-W}^{\infty} g_\sigma(x, \sigma(W))dx = \frac{\underline{W} - W}{\sqrt{2\pi}\sigma^2} e^{-\frac{(\underline{W}-W)^2}{2\sigma^2}} \qquad (25)$$

which has the same sign as $\underline{W} - W$. Thus (**??**) is surely positive for $\underline{W} \geq W$, which means that until $\mathcal{F}$ reaches 50%, $d\mathcal{F}/dW > 0$ is assured. It is easy to check that (**??**) achieves its minimum at $\underline{W} - W = -\sigma$, at which (**??**) has the value $(-1/\sqrt{2\pi e})\sigma$, and (**??**) becomes $d\mathcal{F}/dW = (1 - \sigma_W)/\sqrt{2\pi e})\sigma$. Thus if $\sigma_W < 1$, $d\mathcal{F}/dW$ is always positive, though more cannot be said in general.